

SHOCK WAVE DIFFRACTION AT NORMAL INCIDENCE ON THE FREE SURFACE OF LIQUID CONTAINING A WEDGE WITH ITS TIP AT THE INTERFACE*

K.A. BEZHANOV

Normal incidence of a plane shock wave in gas on the free surface of a compressible liquid, its reflection and diffraction are considered in the case when the liquid occupies a part of the lower half-plane, while the other is taken by a wedge whose tip is at the unperturbed interface. In a particular case one of the wedge sides can coincide with the free surface level line or be at a nearly straight angle to it. Owing to the problem linearity, the flows of liquid and gas are considered separately, viz. the flow of liquid is determined by the pressure behind the shock wave reflected from the solid /wedge/ wall, and the obtained form of the free surface determines the perturbed gas flow /1/.

The problem of irregular interaction between the shock wave and the liquid free surface was considered in /1/, and the shock wave reflection from a corner with a nearly straight tip angle and that of motion of a piston in the form of a dihedral angle close to a straight one were considered in /2/ and /3/, respectively.

1. Statement of the problem. Let a plane shock wave of arbitrary intensity propagate through a quiescent gas and come in contact at the initial instant of time with the free surface of a liquid of considerably higher density than that of gas. A weak diffracted compression wave then propagates through the liquid, and a shock wave is reflected into the gas. If the liquid contains a wedge with its tip at the unperturbed gas-liquid interface, the pattern of interaction between the diffraction zones in the liquid and gas becomes more complex (Fig.1). As the basic unperturbed gas parameters we take the parameters behind the shock wave reflected from the wedge wall, viz. pressure P , density R_1 , and the speed of sound a_1 ; in the liquid we have density R_2 and the speed of sound a_2 /4/. The problem is linear owing to the smallness of parameter $\varepsilon = R_1/R_2$, and pressure perturbations in physical variables (X, Y, t) satisfy the wave equation which, after passing to self-similar coordinates $x = X/(a_1 t)$, $y = Y/(a_1 t)$ ($i = 1, 2$), assume the form of equations

$$(1 - x^2)p_{xx} - 2xyp_{xy} + (1 - y^2)p_{yy} - 2xp_x - 2yp_y = 0 \quad (1.1)$$

which inside the unit circle is of the elliptic type and outside it of the hyperbolic type.

Outside the diffraction region the flow of liquid is piecewise constant, as can be checked by the method of Smirnov-Sobolev /5/. In the diffraction region we obtain a simple inhomogeneous Hilbert problem for an analytic function whose real part represents pressure perturbation. Having determined the pressure, we obtain the form of the liquid free surface.

Outside the diffraction region the gas flow varies in regions bounded by the characteristics issuing from points H and H' , by the shock wave, arcs of the Mach circle, and by the liquid free surface. It is determined by the method of Smirnov-Sobolev. In remaining regions,

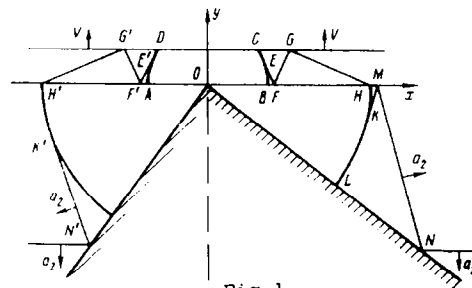


Fig.1

where the perturbations induced by the wedge in the liquid do not propagate, and the flow is determined by the shock wave reflection from the interface of the two media /6/. In the diffraction zone we have the inhomogeneous Hilbert problem for the complex derivative of pressure, whose solution is obtained in explicit form.

Since solution of the problem in gas is determined solely by the second derivative of the free surface form, the notation for these two media can be the same. Branches of the multiple-valued functions that occur below are understood to be those that are positive for positive values of their arguments.

2. Solution in regions occupied by the liquid. The fluid diffraction motion is induced by the wedge from which issue variable perturbations concentrated inside the Mach circle whose center is at the wedge tip. The weak wave that penetrates the liquid is reflected by the wedge walls and is tangent to arcs of the Mach circle for angles of inclination $-\pi/2 < \theta_L \ll -\pi/4$ and $-3\pi/4 < \theta_L' < -\pi/2$ of the right- and left-hand walls, respectively. Pressure behind the reflected wave is equal $2P$. When $-\pi/4 < \theta_L < 0$ ($-\pi < \theta_L' < -3\pi/4$) the reflected wave reaches the free surface and is reflected from it as a rarefaction wave onto the Mach circle arc with a jump of P in conformity with the boundary condition of pressure constancy on the free surface. In the considered here approximation the fronts of compression and rarefaction waves coincide with the characteristics of Eq. (1.1). Thus outside diffraction region the pressure is a piecewise-constant function, and the boundaries of regions of constant pressure pass over weak compression and rarefaction waves at which pressure jumps are equal P . The rarefaction jumps are, obviously, the result of linearization of equations of gasdynamics.

Passing in Eq. (1.1) in regions OLH and $OL'H'$ to polar coordinates (r, θ) and applying Chaplygin's transform

$$r = \frac{2\rho}{1+\rho^2} \quad (2.1)$$

we reduce it to the Laplace equation. We introduce in the plane $\zeta = \rho e^{i\theta} = \xi + i\eta$ the analytic function $\Phi(\zeta) = p + iq$ and the formula for complex velocity /1/

$$u + iv = \frac{1}{2} \int \zeta d\Phi + \frac{1}{\zeta} d\bar{\Phi}, u \rightarrow \frac{u}{a_2}, v \rightarrow \frac{v}{a_2}, p \rightarrow \frac{p}{a_2^2 R_2^2}$$

where φ is a harmonic function conjugate of p , and the arrows imply here and subsequently equality, apart the notation. We now obtain for function $\Phi(\zeta)$ the following boundary value problem in sectors OLH and $OL'H'$

$$p = P, \eta = 0, 0 < \xi < 1 \quad \varphi = 0, \theta = \theta_L, 0 < \rho < 1 \quad (2.2)$$

$$p = [\theta(\theta_1 - \theta) + 1]P, \rho = 1, \theta_L < \theta < 0$$

$$p = P, \eta = 0, -1 < \xi < 0 \quad \varphi = 0, \theta = \theta_L', 0 < \rho < 1$$

$$p = [\theta(\theta - \theta_2) + 1]P, \rho = 1, -\pi < \theta < \theta_L' \quad (2.3)$$

where θ_1 and θ_2 are angles of tangency of reflected compression or rarefaction waves that correspond to points K and K' , and θ is the Heaviside function. Below we present the solutions of boundary value problems in combined form with the superscript and $j=1$ corresponding to sector OLM , and the subscript and $j=2$ to sector $OL'M'$.

Function

$$\omega = \pm \left(\frac{1 - (\pm \zeta)^{n_j}}{1 + (\pm \zeta)^{n_j}} \right)^2, \quad n_1 = -\frac{\pi}{\theta_L}, \quad n_2 = \frac{\pi}{\pi + \theta_L'}$$

maps sectors OLM and $OL'M'$ onto the upper half-plane and the substitution $\Phi(\omega) = \sqrt{1 \mp \omega}$ $\Psi(\omega)$ enables us to reduce (2.2) and (2.3) to the Dirichlet problem for $\text{Re}\Psi(\omega)$. When $-\infty < \sigma < +\infty$

$$\text{Re}\Psi(\omega) = \{2\theta(\mp \sigma \pm \sigma_j) + \theta[(1 \mp \sigma)(\pm \sigma \mp \sigma_j)]\} \frac{P}{\sqrt{1 \mp \omega}}$$

$$\sigma_j = \mp \text{tg}^2 \alpha_j, \quad \alpha_1 = -\frac{\pi \theta_1}{2\theta_L}, \quad \alpha_2 = \frac{\pi(\pi + \theta_2)}{2(\pi + \theta_L')}$$

where σ_j correspond to points K and K' . Representing the solution of the Dirichlet problem in terms of the Schwarz integral we finally obtain

$$\Phi(\omega) = P \mp \frac{iP}{\pi} \ln \frac{\sqrt{1 \mp \sigma_j} - \sqrt{1 \mp \omega}}{\sqrt{1 \mp \sigma_j} + \sqrt{1 \mp \omega}}, \quad \Phi(0) = P$$

Distribution of the vertical velocity component at the interface and the shape of free surface $y = f(x)$ are determined using the following differential equations

$$\begin{aligned} v'(x) &= -x^{-1} \sqrt{1-x} \varphi'(x), \quad v(\pm 1) = -v_0 \\ x f'(x) - f(x) &= -v(x), \quad f(\pm 1) = -y_0 \end{aligned} \quad (2.4)$$

$$\varphi(x) = \pm \frac{P}{\pi} \ln \frac{1 + (\pm \xi)^{n_j} + 2 \cos \alpha_j (\pm \xi)^{n_j/2}}{1 + (\pm \xi)^{n_j} - 2 \cos \alpha_j (\pm \xi)^{n_j/2}}, \quad x = \frac{2\xi}{1 + \xi^2}$$

From (2.4) we obtain asymptotic formulas for $v(x)$ and $f(x)$ in the coordinate origin neighborhood with $\theta_L \neq -\pi/4$ and $\theta_L \neq -3\pi/4$

$$v(x) \sim v(0) - \frac{4Pn_j \cos \alpha_j}{\pi(n_j - 2)2^{n_j/2}} (\pm x)^{n_j/2 - 1}, \quad n_j > 2 \quad (2.5)$$

$$f(x) \sim f(0) + \frac{8Pn_j \cos \alpha_j}{\pi(n_j - 2)(n_j - 4)2^{n_j/2}} (\pm x)^{n_j/2 - 1}$$

and for $\theta_L = -\pi/4$ and $\theta_L = -3\pi/4$ we have the solution of problem (2.4)

$$\begin{aligned} v(x) &= v(0) \mp 2\pi^{-1} P \arcsin x, \quad v(0) = f(0) = P - v_0 \\ f(x) &= f(0) \pm 2\pi^{-1} P (x \ln |x| - x \ln(1 + \sqrt{1-x^2}) - \arcsin x) \end{aligned} \quad (2.6)$$

In conformity with (2.5) and (2.6) there is always a rise of points of the free surface over the unperturbed level in the neighborhood of the coordinate origin, which can be explained by the compressing action of waves reflected from the wedge walls. When the wedge is positioned symmetrically relative to the Oy axis, function $f(x)$ at point $x=0$ is continuous and $f'(x)$ is discontinuous, while in the unsymmetric case $f(x)$ is discontinuous at point $x=0$. When $-\pi/4 < \theta_L < 0$ and $-\pi < \theta_L < -3\pi/4$ the left- and right-hand derivatives $f'(\pm 0)$ are bounded, and when $-\pi/2 < \theta_L \leq -\pi/4$ and $-3\pi/4 \leq \theta_L < -\pi/2$, we have $f'(\pm 0) = -\infty$, which is explained by the considerable pushing effect of reflected waves. Hence $f''(x)$ can contain in the symmetric case singular generalized functions of the form $\delta(x)$, $(-x)_-^\alpha$ and x_+^α , in the unsymmetric case of the form $\delta'(x)$, $(-x)_-^\alpha$ and x_+^α , where $\alpha > -2$, $\delta(x)$ is the delta function and $\delta'(x)$ its derivative, and $(-x)_-^\alpha$, x_+^α represent the regularization of functions with power singularities $/8/$.

3. The flow of gas outside the diffraction region. Transform $\mu = \arccos r^{-1}$, $r > 1$ reduces Eq. (1.1) to the wave equation $p_{\mu\mu} - p_{\theta\theta} = 0$ whose characteristics are half-tangents to the Mach circle and oriented in various directions $/5/$. To formulate boundary conditions we pass to the system of coordinates $Ox'y'$ turned by angle $\pi/2$ relative to Oxy (in what follows, primes at x' and y' are omitted).

In regions FHG and $F'H'G'$ we seek a solution of the form $p = \chi_j(\mu \pm \theta)$ ($j = 1, 2$), where the superscript and the first index relate here and subsequently to region $BHGC$ and the subscript to region $AH'G'D$. The arbitrary functions χ_j are determined using the conditions at the interface

$$p_x(0, y) = y^2 f''(-y), \quad y_F < y < y_H, \quad y_{F'} < y < y_{H'} \quad (3.1)$$

where $f(-y)$ is the gas-liquid interface, and when one of the wedge walls is at a small angle to the unperturbed free surface or coincides with it, it also defines the gas-wall interface.

The final solution is of the form

$$p = \int_{\mu_j \mp \pi/2}^{\mu \pm \theta} \operatorname{cosec}^2 \lambda f''(\mp \operatorname{cosec} \lambda) d\lambda \quad \begin{aligned} \mu_1 &= \arccos y_H^{-1} \\ \mu_2 &= \arccos y_{H'}^{-1} \\ y_H &= \dots = y_{H'} = a_2 |a_1| \end{aligned}$$

In regions $CEFG$ and $DE'F'G'$ we seek a solution of the form $p = \chi_l(\mu \pm \theta) + \kappa_l(\mu \mp \theta)$ ($l = 3, 4$), where functions χ_l are known from the solution in regions FHG and $F'H'G'$, and κ_l is determined by the condition at the shock wave

$$\begin{aligned}
 m_1^2 p_x + [mBy^{-1} + (m+A)y]p_y &= 0, \quad x = m \\
 A &= \frac{M^2 + 1}{2mM}, \quad B = \frac{\gamma + 1}{2} \frac{M^2 - 1}{(\gamma - 1)M^2 + 2}, \quad M = \frac{U + V}{a_0}, \\
 m &= \frac{U}{a_1}, \quad m_1^2 = 1 - m^2
 \end{aligned} \tag{3.2}$$

where U is the velocity of the shock wave reflected from the /wedge/ solid wall, V and a_0 are, respectively, the stream velocity and the speed of sound behind the incident shock wave, and γ is the specific heat ratio.

The final solution is of the form

$$p = \int_{\mu_j \mp \pi/2}^{\mu_j \mp \theta} \operatorname{cosec}^3 \lambda f''(\mp \operatorname{cosec} \lambda) d\lambda + \int_{\mu_1 \mp \pi/2}^{\mu_1 \mp \theta} \frac{C(\lambda) + D(\lambda)}{C(\lambda) - D(\lambda)} g^3(\lambda) f''(\pm g(\lambda)) d\lambda \tag{3.3}$$

$$\begin{aligned}
 C(\lambda) &= (1 - m \cos \lambda)[m(m_1^2 + B)\sin^2 \lambda - (m + A)(1 - m \cos \lambda)^2] \\
 D(\lambda) &= (m - \cos \lambda)[(1 + mA)(1 - m \cos \lambda)^2 - m^2 B \sin^2 \lambda] \\
 g(\lambda) &= m_1^{-2} \operatorname{cosec} \lambda (1 + m^2 - 2m \cos \lambda) \\
 \mu_3 &= \arccos y_F^{-1}, \quad \mu_4 = \arccos y_F^{-1}
 \end{aligned}$$

In regions BFE and $AF'E'$ the solution is of the form (3.3).

The form of the shock wave $x = m + \psi(y)$ in sections CG and DG' is determined by the solution of the Cauchy problem for $x = m$

$$y\psi'(y) - \psi(y) = -BM_1^{-1}p(y), \quad \psi(y_j) = BM_1^{-1}p(y_j) \quad (j = 1, 2)$$

where $p(y_1) = p(y_2)$ is taken from /6/ with $y_1 = y_C$, $y_2 = y_{C'}$, $M_1 = V/a_1$.

4. The flow of gas in the diffraction region. The diffraction region is bounded by the shock wave, two arcs of the Mach circle, the liquid free surface and, possibly, by one of the wedge walls. The condition at the interface is of the form (3.1) for $-1 < y < 1$, and the presence of multiplier y^2 cancels all singularities at $f''(-y)$. For $-m_1 < y < m_1$ the condition at the shock wave is of the form (3.2).

The boundary condition at arcs of the Mach circle, obtained from (3.3), is

$$p_\theta(1, \theta) = c(\theta), \quad -\pi/2 < \theta < \theta_C, \quad \theta_D < \theta < \pi/2$$

where

$$c(\theta) = \operatorname{cosec}^3 \theta f''(-\operatorname{cosec} \theta) + \frac{C(\theta) + D(\theta)}{C(\theta) - D(\theta)} g^3(\theta) f''(-g(\theta))$$

Besides it, two integral conditions which specify smoothness of the shock wave front at points C and D and the pressure change along CD by the given quantity

$$\int_{-m_1}^{m_1} \frac{p_y}{y} dy = -\frac{mM_1}{B} (\psi_y(-m_1) - \psi_y(m_1)), \quad \int_{-m_1}^{m_1} p_y dy = p_D - p_C \tag{4.1}$$

must be satisfied. In (4.1) $\psi_y(m_1)$, $\psi_y(-m_1)$, p_C , p_D are known from the solution in regions $BHGC$ and $ADG'H'$, and p_C and p_D represent the pressure at points C and D .

After passing to polar coordinates and application of transform (2.1), Eq.(1.1) assumes the form of the Laplace equation. The diffraction regions are represented by a curvilinear orthogonal quadrangle of the ζ plane, bounded by arcs of circle $\{\rho = \rho(\theta), -\theta_1 < \theta < \theta_1\}$, $\{\rho = 1, -\pi/2 < \theta < -\theta_1\}$, $\{\rho = 1, \theta_1 < \theta < \pi/2\}$ and the straight line segment $\{\xi = 0, -1 < \eta < 1\}$. Boundary conditions for the normal and tangent derivatives of pressure are of the form

$$\begin{aligned}
 ap_n + bp_s &= c \\
 a &= a(\theta), \quad b = b(\theta), \quad c = 0, \quad \rho = \rho(\theta), \quad \theta_C < \theta < \theta_D \\
 a &= 0, \quad b = 1, \quad c = c(\theta), \quad \rho = 1, \quad -\pi/2 < \theta < \theta_C \\
 a &= 0, \quad b = 1, \quad c = c(\theta), \quad \rho = 1, \quad \theta_D < \theta < \pi/2 \\
 a &= 1, \quad b = 0, \quad c = c_0(\theta), \quad \xi = 0, \quad -1 < \eta < 1
 \end{aligned} \tag{4.2}$$

where

$$\begin{aligned}
 a(\theta) &= \sqrt{1 - m^2 \sec^2 \theta}, \quad b(\theta) = B \operatorname{ctg} \theta - mA \operatorname{tg} \theta, \\
 \theta_1 &= \arccos m \\
 c_0(\eta) &= \frac{8\eta^2}{(1 + \eta^2)^2} f''\left(-\frac{2\eta}{1 + \eta^2}\right), \quad \rho(\theta) = \frac{\cos \theta - \sqrt{\cos^2 \theta - m^2}}{m}
 \end{aligned}$$

n is the normal, and the orientation of (n, s) coincides with that of (x, y) .

Let us map the region outlined by the curvilinear quadrangle of the ζ plane onto the interior of the rectangle $\{0 < \sigma < \sigma_0, 0 < \tau < \pi\}$ of the plane $\omega = \sigma + i\tau$

$$\omega = \ln \frac{1+\zeta}{1-\zeta} + i \frac{\pi}{2}, \quad \sigma_0 = \frac{1}{2} \ln \frac{1+m}{1-m}$$

and introduce the analytic function $W(\omega) = p_\sigma - ip_\tau$.

The boundary condition (4.2) then assumes the form of the Hilbert problem

$$\begin{aligned} a_1 p_\sigma - b_1 p_\tau &= d \\ a_1 &= a_1(\tau), \quad b_1 = b_1(\tau), \quad d = 0, \quad \sigma = \sigma_0, \quad 0 < \tau < \pi \\ a_1 &= 1, \quad b_1 = 0, \quad d = d_-(\sigma), \quad \tau = 0, \quad 0 < \sigma < \sigma_0 \\ a_1 &= 1, \quad b_1 = 0, \quad d = d_+(\sigma), \quad \tau = \pi, \quad 0 < \sigma < \sigma_0 \\ a_1 &= 1, \quad b_1 = 0, \quad d = d_0(\tau), \quad \sigma = 0, \quad 0 < \tau < \pi \end{aligned}$$

where

$$\begin{aligned} a_1(\tau) &= \frac{1}{2} \sin 2\tau, \quad b_1(\tau) = \frac{m}{m_1^2} B - A \cos^2 \tau, \\ d_3(\tau) &= \cos^2 \tau f''(\cos \tau) \\ d_\mp(\sigma) &= \mp \operatorname{ch}^2 \sigma f''(\pm \operatorname{ch} \sigma) \mp \frac{C_1(\sigma) + D_1(\sigma)}{C_1(\sigma) - D_1(\sigma)} \times \frac{\operatorname{ch}^3(2\sigma_0 - \sigma)}{\operatorname{ch} \sigma} f''(\pm \operatorname{ch}(2\sigma_0 - \sigma)) \\ C_1(\sigma) &= \operatorname{ch}(\sigma_0 - \sigma)[m(m_1^2 + B) - m_1^2(m + A)\operatorname{ch}^2(\sigma_0 - \sigma)] \\ D_1(\sigma) &= \operatorname{sh}(\sigma_0 - \sigma)[m_1^2(1 + mA)\operatorname{ch}^2(\sigma_0 - \sigma) - m^2 B] \end{aligned}$$

In the new variables conditions (4.1) assume the form

$$\int_0^\pi \sec \tau p_\tau d\tau = \frac{M_1}{B} (\psi_y(-m_1) - \psi_y(m_1)), \quad \int_0^\pi p_\tau d\tau = p_D - p_C \tag{4.3}$$

We map the rectangle of plane ω onto the upper half-plane of plane w , using function

$$w = \frac{\vartheta_2(0, q) \vartheta_2(-i\omega, q)}{\vartheta_3(0, q) \vartheta_3(-i\omega, q)}, \quad q = \frac{1-m}{1+m}$$

where $\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4$ are elliptic theta functions [9]. The intervals $(-\infty, -1)$ and $(1, +\infty)$ correspond to the shock wave, and $0 < k < 1$ where k is the modulus of the elliptic integral, corresponds to the wall $(-k, k)$. The index of the obtained Hilbert problem with discontinuous coefficients in the class of functions integrable at points $w = \pm 1$, is equal unity.

To determine the canonical function we represent it in the form [2,3,10/

$$Z(w) = Z_1(w)Z_2(w)$$

where

$$Z_1(w) = \frac{1}{\sqrt{w^2 - 1}} = i \frac{\vartheta_3(0, q) \vartheta_3(-i\omega, q)}{\vartheta_4(0, q) \vartheta_4(-i\omega, q)}$$

has a piecewise constant argument at the boundary and eliminates discontinuities at points $w = \pm 1$, and $Z_2(w)$, satisfies the condition on the image of the shock wave, and is of the form [2/

$$\begin{aligned} Z_2(w(\omega)) &= \exp \left(- \sum_{n=1}^{\infty} \frac{2 - e^n - h^n}{n \operatorname{sh} 2n\sigma_0} \operatorname{ch} 2n\omega \right) \\ e &= \frac{l_1 - 1}{l_1 + 1}, \quad h = \frac{l_2 - 1}{l_2 + 1}, \quad \frac{1}{l_1 + l_2} = A - \frac{mB}{m_1^2}, \\ \frac{l_1 l_2}{l_1 + l_2} &= \frac{mB}{m_1^2} \end{aligned}$$

Solution of the Hilbert problem has then the form [11,12/

$$W(\omega(w)) = \frac{Z(m)}{\pi i} \left(\int_{-1}^{-k} \frac{d_+(s)}{Z(s)} \frac{ds}{s-w} + \int_{-k}^k \frac{d_0(\tau(s))}{Z(s)} \frac{ds}{s-w} + \int_k^1 \frac{d_-(s)}{Z(s)} \frac{ds}{s-w} \dots A_0 - A_1 w \right) \tag{4.4}$$

where the imaginary constants A_0 and A_1 are determined by conditions (4.3).

Having determined pressure distribution at the shock wave front, using (4.4), we obtain its form on section CD with $(-m_1 < y < m_1)$ from the solution of the Cauchy problem

$$y\psi'(y) - \psi(y) = -BM_1^{-1}p(y), \quad \psi(m_1) = \psi_0$$

while the fulfillment of conditions (4.3) ensures the continuity of $\psi(y)$ at point D and smoothness at points C and D .

The pressure distribution at the interface and arcs of the Mach circle which are obtained by integrating the limit values (4.4), has no singularities, as in /2/. The absence of singularities is mathematically explained by the shift-free argument of the second derivative of the interface form, owing to which the right-hand side of the boundary condition (3.1) is a regular generalized function in the neighborhood of the wedge tip. The absence of shift of the second derivative of the interface is related to the absence of a stream parallel to the unperturbed level of the liquid free surface. Such stream induces pressure peaks at points of break and discontinuity of the boundary, which give rise to singularities in linear problems /1,3,10/. The determination of pressure distribution at the interface enables us to find the solution in the region occupied by the liquid, and to take into account the effect of flows on both sides of the wedge.

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